1 stepped pressure equilibrium code : ph00aa

Contents

1.1 outline

1. Pressure-jump Hamiltonian o.d.e.'s, and tangent map,

1.1.1 Pressure jump Hamiltonian

2. The pressure-jump Hamiltonian is derived from the force-balance condition at the ideal interfaces. Let p_1 and $\mathbf{B}_1$ be the pressure and field immediately inside the interface, and p_1 and $\mathbf{B}_1$ be the pressure and field immediately outside, then the force balance condition requires that

$$H \equiv 2\,\delta p = 2(p_1 - p_2) = B_2^2 - B_1^2 \tag{1}$$

For Beltrami fields, which satisfy $\nabla \times \mathbf{B} = \mu \mathbf{B}$, the magnitude of the field, B, on the interface (where we assume that $B^s = 0$) may be written

$$B^2 = \frac{g_{\phi\phi} f_\theta f_\theta - 2 g_{\theta\phi} f_\theta f_\phi + g_{\theta\theta} f_\phi f_\phi}{g_{\theta\theta} g_{\phi\phi} - g_{\theta\phi} g_{\theta\phi}} \tag{2}$$

where f is a surface potential and $g_{\theta\theta}$, $g_{\theta\phi}$ and $g_{\phi\phi}$ are metric elements local to the interface.

3. Assuming that the field B_1 is known on the 'inside' of the interface, ie. $B_{1\theta} = f_\theta$, $B_{1\phi} = f_\phi$ and f is known, it is required to determine the tangential field, $p_\theta = B_\theta$ and $p_\phi = B_\phi$, on the 'outside' of the interface.

4. The o.d.e.'s are given by Hamilton's equations

$$\dot\theta = \frac{\partial H}{\partial p_\theta}\Big|_{\theta,\phi,p_\phi}, \quad \dot p_\theta = -\frac{\partial H}{\partial \theta}\Big|_{p_\theta,\phi,p_\phi}, \quad \dot\phi = \frac{\partial H}{\partial p_\phi}\Big|_{\theta,p_\theta,\phi}, \quad \dot p_\phi = -\frac{\partial H}{\partial \phi}\Big|_{\theta,p_\theta,p_\phi}, \tag{3}$$

where the 'dot' denotes derivative with respect to 'time'.

5. This is reduced to a $1\frac{1}{2}$ dimensional system by using ϕ as the time-like integration parameter, and replacing the equation for $\dot p_\phi$ with

$$p_\phi = P(\theta, p_\theta, \phi; \delta p) = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{4}$$

where $a = g_{\theta\theta}$, $b = -2 g_{\theta\phi} p_\theta$ and $c = g_{\phi\phi} p_\theta^2 - b_1 - 2\,\delta p\, G$ (see below for definition of b_1 and G).

6. The o.d.e.'s that then need to be followed are (see below for definition of A and b_2)

$$\frac{d\theta}{d\phi} = \frac{g_{\phi\phi} p_\theta - g_{\theta\phi} p_\phi}{-g_{\theta\phi} p_\theta + g_{\theta\theta} p_\phi}, \tag{5}$$

$$\frac{dp_\theta}{d\phi} = \frac{g_{\phi\phi,\theta}(f_\theta^2 - p_\theta^2) - 2 g_{\theta\phi,\theta}(f_\theta f_\phi - p_\theta p_\phi) + g_{\theta\theta,\theta}(f_\phi^2 - p_\phi^2) + A + (b_2 - b_1) G_{,\theta}/G}{-2 g_{\theta\phi} p_\theta + g_{\theta\theta} 2 p_\phi}. \tag{6}$$

7. Note that $d\theta/d\phi = B^\theta/B^\phi$; there is a fundamental relation between the pressure-jump Hamiltonian and the field-line Hamiltonian. (Furthermore, in many cases the surface will be given in straight field line coordinates, so $d\theta/d\phi = const.$.)

8. The conserved 'energy' δp is given by the jump in pressure across the interface.

9. Some description of the internal notation follows :

$$G = g_{\theta\theta} g_{\phi\phi} - g_{\theta\phi} g_{\theta\phi}, \tag{7}$$

$$A = g_{\phi\phi} 2 f_\theta f_{\theta\theta} - 2 g_{\theta\phi} (f_{\theta\theta} f_\phi + f_\theta f_{\phi\theta}) + g_{\theta\theta} 2 f_\phi f_{\phi\theta}, \tag{8}$$

$$b_1 = g_{\phi\phi} f_\theta^2 - 2 g_{\theta\phi} f_\theta f_\phi + g_{\theta\theta} f_\phi^2, \tag{9}$$

$$b_2 = g_{\phi\phi} p_\theta^2 - 2 g_{\theta\phi} p_\theta p_\phi + g_{\theta\theta} p_\phi^2, \tag{10}$$

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